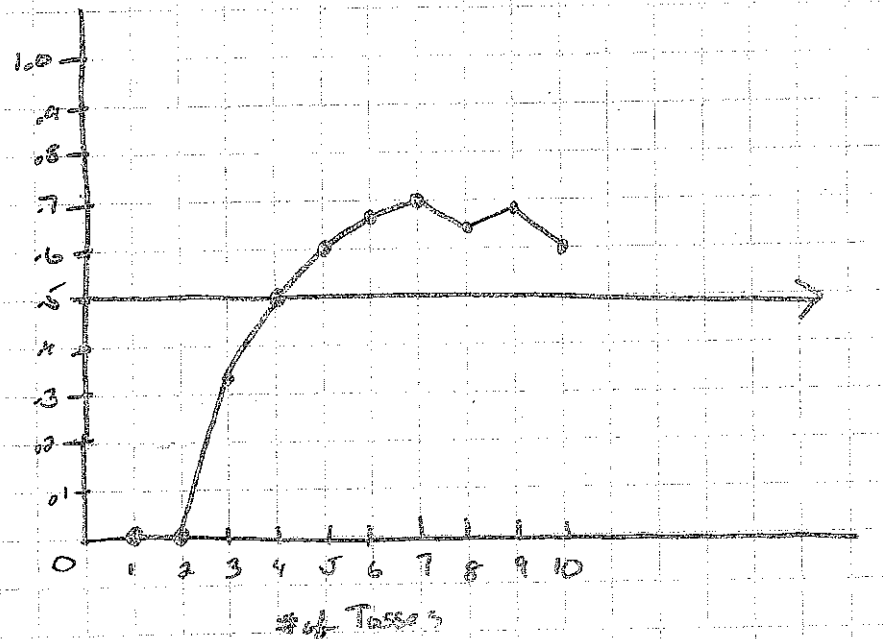
Chapter 6: Probability: The Study of Randomness

6.1 The Idea of Probability

Random - Chance behavior is unpredictable in the short run but has a regular and predictable pattern in the long run. (Individual outcomes are uncertain)

Probability - The proportion of times the outcome would occur in a very long series of repetitions.

Have a student flip a coin n times.
Keep track of the order of H or T.
Find prob. after each toss.
Have a class make a conjecture.



Prob Flipping "Tails"

1 - H 0/1 = 0
2 - H 0/2 = 0
3 - T 1/3 = .33
4 - T 2/4 = .5
5 - T 3/5 = .6
6 - T 4/6 = .67
7 - T 5/7 = .7
8 - H 5/8 = .625
9 - T 6/9 = .67
10 - H 6/10 = .6

Outcomes For Dice: 1, 2, 3, 4, 5, 6
Deck of Cards

$P(\spadesuit \text{ or } \heartsuit)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
ADDITION RULE

Counting Principle
Tree Diagram
Sample Space
Marbles in an urn (w/ + w/out replacement)

$P(R \text{ and } B) = P(R)\,P(B)$ M.U. Rule
$P(R \text{ or } B) = P(R) + P(B)$ Add. Rule for Disjoint events

$P(\text{snow}) = .1 \quad \therefore P(\text{not snow}) = 1 - .1 = \boxed{.9}$

$P(A^c) = 1 - P(A)$

$0 \leq P(A) \leq 1$

6.2 Probability Models

OBJ: You will solve problems requiring probability Rules

Class 6.11, 6.15, 6.17, Ex 6.10 pg 345, 6.20, 6.21, 6.23
Ex: 6.12, 6.26, 6.28, 6.30

Hw: 6.40 - 6.44 (Δ 6.41)

$P(A \text{ or } B) = P(A \cup B)$

$P(A \text{ and } B) = P(A \cap B)$

6.3 General Probability Rules - Day 1

OBJ: You will find the probabilities of conditions using Venn Diagrams.

Union - The union of any collection of events is the event that at least one of the collection occurs.

$0 \leq P(A) \leq 1$

$P(S) = 1$

$P(A^c) = 1 - P(A)$: A^c = compliment (Give an example of a complementary event)

Disjoint: When two events A and B have no outcomes in common and so can never occur simultaneously. (Also called Mutually Exclusive) (Disjoint events are not independent).

$P(A \text{ or } B) = P(A) + P(B)$

$P(K \text{ or } 7) = P(K) + P(7)$

NOT DISJOINT: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ → This works for Disjoint Also b/c $P(A \text{ and } B) = 0$

Independent: $P(A \text{ and } B) = P(A)\,P(B)$

$\cup$ = Union = OR

$\cap$ = Intersection = AND (Joint Prob)

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ ← Equiv

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ * on FS ←

Ex 6.17
6.46
6.47

HW: 6.48, 6.49, 6.50

6.3 General Probability Rules - Day 2

OBJ: You will calculate conditional probabilities

$P(A|B)$ The prob. of A given we know the prob. of B.
The prob. of A given the prob. of B.

$P(A|B) = \frac{P(A \text{ AND } B)}{P(B)} = \frac{P(A \cap B)}{P(B)}$ * on F.S.

A and B are indep. when:

$P(A|B) = P(A)$ → B has no influence on A } $P(A|B) = P(A)$
$P(B|A) = P(B)$
$P(X|Y) = P(X)$
Etc.

$P(A \text{ AND } B) = P(A) \times P(B|A)$
$= P(A) \times \frac{P(B \text{ AND } A)}{P(A)}$ } JOINT PROB - Does not show indep.
- They are always =

$P(A \text{ AND } B) = P(B \text{ AND } A)$

- This just checks work to see if it is correct.

Multiplication Rule
$P(A \text{ AND } B) = P(A) \cdot P(B)$
$P(A \text{ AND } B) = P(A) \cdot P(B|A)$ ← conditional →

Independent
$P(A \text{ AND } B) = P(A) \cdot P(B)$
$P(A|B) = P(A)$

Example 6.21
6.67
6.69

What is the probability of rolling a die 6 times and rolling:
a) one 5: ${}_6C_1 (1/6)^1 (5/6)^5$
b) At most two 3's: ${}_6C_2 (1/6)^2 (5/6)^4 + {}_6C_1 (1/6)^1 (5/6)^5 + {}_6C_0 (1/6)^0 (5/6)^6$
c) At least one 2: ${}_6C_1 (1/6)^1 (5/6)^5 + {}_6C_2 (1/6)^2 (5/6)^4 + \ldots + {}_6C_6 (1/6)^6 (5/6)^0$ ←

- OR -

$P(0) + \boxed{P(1) + P(2) + P(3) + P(4) + P(5) + P(6)} - P(0)$
$= 1 - P(0)$
$= 1 - {}_6C_0 (1/6)^0 \cdot (5/6)^6$ ←

${}_nC_r\, p^r q^{n-r} \Rightarrow \underbrace{\binom{n}{k} p^k (1-p)^{n-k}}_{\text{on F3}}$

HW: 6.54, 6.55