Chapter 7: Random Variables
7.1 Discrete and Continuous Random Variables - Day 1
OBJ: You will answer questions after setting up a probability distribution table.

Random variable - A variable whose value is a numerical outcome of a random phenomenon.
(Notation: Capital letter @ End of Alphabet: X, Y)

Toss a coin twice. Let X be the # of heads.
If we get
H, T, X=1
H, H, X=2
T, H, X=1
T, T, X=0
Construct a probability Distribution Table Below

Probability Distribution - Lists X and their probabilities.

Value of X :	X_1	X_2	X_3	X_4	X_K
Probability :	P_1	P_2	P_3	P_4	P_K

$0 \leq p_i \leq 1$
$p_1 + p_2 + p_3 + \ldots + p_k = 1$

So for the coin problem the prob. dist. table is:

X	0	1	2
Prob:	1/4	1/2	1/4

The possible values for X are 0, 1, or 2.
As you continue to flip, X may become another value.
(X varies w/each toss ∴ X is a random variable)

$\overline{X}$ (Mean) is the greatest interest to us.

Discrete Random Variable - when X has a countable number of possible values.

Example 7.1 - Read to class - Have class set up a prob. dist.

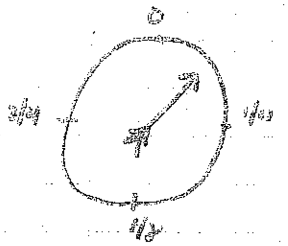
Prob. Histogram - Top of pg 393
- Heights show the prob. of the base
- Heights ADD to 1 (They are prob)
- Bars have equal widths
- Area of Bar = Prob. of outcome

Class: 7.1 7.2, 7.3

HW: 7.15, 7.16, Read pgs 397-401

7.1 Discrete & Continuous Random Variables – Day 2

OBS: You will find the probabilities of continuous random variables.



$P(.3 \le x \le .7)$

.3	.4	.5	.6	.7	How About	.31	.32	.33	...	.69
1/10	1/10	1/10	1/10	1/10		$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	...	1/100

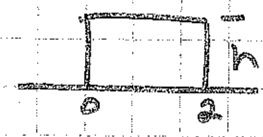
∴ Can't Assign indiv. prob. and then add!! :(

Instead, we use areas under a density curve (Chp. 2) to assign probabilities!! :)

Why? ⇒ Area under a density curve = 1

Corresponding to a total prob of 1. Chp 2 $P\left(Z = \frac{x-\mu}{\sigma}\right)$

Uniform Distribution



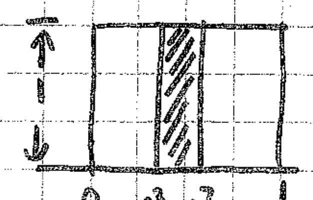
h = ?

Continuous Random variable – when x takes All values in an interval of numbers. (Ex: Depth @ various pts in Lake Erie in feet).

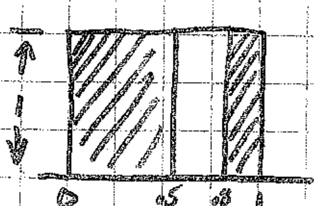
The prob. dist. of x is described by a Density Curve.

The prob of any event is the area under the density curve that make up that event.

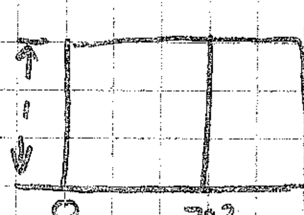
* The prob. dist. assign every indiv. outcome a prob of 0.
(Why? – A pt. has no dimensions)



$P(.3 \le x \le .7)$
$A = lw = (1)(.4)$
$= .4$



$P(x \le .5 \text{ or } x \ge .8)$
$A = lw + lw$
$= (1)(.5) + (1)(.2)$
$= .7$



$P(x = .723)$
$A = lw$
$= 1(0)$
$= 0$

Now ∴ $P(.3 \le x \le .7) = P(.3 < x < .7)$

Go back to spinner and find $P(x = 1/4)$

⟹

Normal Distributions (Table A, normalcdf) are probability distributions.

$N(\mu, \sigma)$

$z = \frac{x-\mu}{\sigma}$ } How many std. dev. the orig. obs. is above (+) or below (−) the mean

$\hat{p}$ = sample proportion
p = Population proportion

so... $z = \frac{x-\mu}{\sigma}$ becomes $z = \frac{\hat{p}-p}{\sigma}$ when we are working with proportions !! :)

7.8

<u>HW</u>: 7.6, 7.7, 7.17, 7.20

7.2 Means + Variables of Random Variables - Day 1

<u>OBJ:</u> You will find the expected value (μ_x) of a set of obs.

Example 7.5

Payoff X	\$0	\$500
Prob(x)	.999	.001

$\mu_x = 0(.999) + 500(.001)$
$= .50$

$\bar{X}$ = The mean of the <u>Actual</u> amount you win

μ_x = The Long-run <u>AVG</u> winnings you expect if you play a very large number of times (Expected Value)

<u>Mean of a Discrete Random Variable:</u>

Value of x	X_1	X_2	X_3 ..... X_k
Prob (x)	P_1	P_2	P_3 ..... P_k

$\mu_x = X_1 p_1 + X_2 p_2 + X_3 p_3 + \dots + X_k p_k$

$\boxed{\mu_x = \sum X_i p_i}$ *on F.S.

Example 7.7

<u>Variance of a Discrete Random Variable:</u> $\boxed{\sigma_x^2 = \sum (X_i - \mu_x)^2 p_i}$ * on F.S

<u>St. Dev of a Discrete Random Variable:</u> $\sigma_x = \sqrt{\text{Variance}}$

7.22
7.24
7.25

<u>Law of Large Numbers</u> - pg 413

Have students write down 10 possible outcomes for a coin toss...
then

<u>Law of Small numbers</u> - pg 415/416

7.32
7.33

<u>HW</u>: 7.29a (Use example 7.7), Look @ pgs. 418-424

7.2 Means + Variance of Random Variables - Day 2

OBJ: You will apply the rules for means + variances.

From pg. 418

Rules for Means

Rule 2: New Refrigerators can have Dimples (X) or paint sags (Y)

$X = 0, 1, 2, \ldots$ } Suppose { $\mu_x = .7$

$Y = 0, 1, 2, \ldots$ } { $\mu_y = 1.4$

The Total # of Defects is $x+y$

Then, the Avg. # of Defects is $\mu_x + \mu_y = \mu_{x+y} = 2.1$

$\therefore \boxed{\mu_{x+y} = \mu_x + \mu_y}$ ⟹ The mean of a sum of rand. vars. is always the sum of their means.

pg 418 #3

Rule 1:

L_1	$L_2 = 5L_1 + 3$
1	8
2	13
3	18
Avg = 2 ⟶	Avg = 5 (Avg L_1) + 3 = 5(2) + 3 = 13

If you have L_1 there is no need to create L_2 if all you're interested in is the new mean... Just work with the old mean.

$$\boxed{\mu_{a+bx} = a + b\mu_x}$$

Rules for Variances

Rule 1: x is random, a + b are fixed: $\boxed{\sigma^2_{a+bx} = b^2\sigma_x^2}$

Think: $\sigma_{a+bx} = b\sigma_x$

- Adding a constant to a set of obs. does not Δ σ_x
- Mult. a constant Δ's the variance by b^2!

so.... if you multiply by 2, the variance gets mult. by 4.

(Mult. by 3 - Var by 9, Var by 25 - Mult. by 5)

Rule 2: X + Y are independent random vars.

$$\boxed{\sigma^2_{x+y} = \sigma_x^2 + \sigma_y^2} \qquad \boxed{\sigma^2_{x-y} = \sigma_x^2 + \sigma_y^2}$$

Why ADD when σ^2_{x-y}:

$\sigma^2_{x-y} = \sigma^2_{x+(-1)y}$

$= \sigma_x^2 + (-1)^2\sigma_y^2$

$= \sigma_x^2 + \sigma_y^2$:)

*** Variances ADD, St. Dev Do NOT !!

Example 7.11

7.34, 7.35, 7.36, 7.39 → show $\sqrt{2^2 + 1^2} \neq 2 + 1$

HW: 7.37 (Don't Do Last Part), 7.38 (1st Sent.), 7.45, 7.62 b, c

Day 3:

Class: 7.46, 7.63 C